

# More Detailed Descriptions of Locality and Realism

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Many experiments have shown that locality-realism theory is at variance with quantum mechanics predictions. Although locality and realism, which are two different conceptions, are given respective definition, the descriptions of the both are a little of abstract when they are applied to real experimental situations. The abstract descriptions result in difficulty for one to judge whether the variance come from locality or realism or both. Here we provide more detailed descriptions of locality and realism, and show that any system being in a pure state or a non-maximally mixed state has property of non-realism. We also present experimental schemes feasible under current technologies to test the non-locality realism. The connections between non-locality and entanglement and correlation are also discussed.

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The local realism theory (LRT) has been proven to be true in classical word. In quantum word, however, LRT is at variance with quantum mechanics predictions. This variance was pointed out first by Einstein, Podolsky and Rosen (EPR) [1]. But EPR think LRT should be accepted, and the theory of quantum mechanics (QM) is incomplete. They believe that a complete theory of QM is possible. later, a possible complete theory of QM, hidden-variable model (HVM), was put forward. In 1964, Bell shown that the value of a certain combination of correlation on two distant systems cannot be higher than a value if we accept LRT and the HVM [2]. Bell's theorem provided a possibility for one to judge experimentally whether QM is a LRT added by the HVM or not. Later, some improved versions of Bell's theorem have been put forward, such as CHSH inequality and so on [3, 4, 5, 6]. Greenberger, Horne and Zeilinger (GHZ) [7] present a theorem without inequality which showed that a certain correlation of the quantum systems may have conflict with LRT. There are many different versions of GHZ theorem, such as some versions in Refs.[8, 9, 10].

LRT has two assumptions [1, 11]: realism and locality. Realism claims that all measurement outcomes depend on pre-existing properties of the system that are independent of the measurement while locality claims that there are no influences between events in space-like separated systems. Up to now, all experiments motivated by these Bell' theorems are completely consistent with QM's predictions [12, 13, 14, 15], and so deny the LRT in quantum word. The failure of LRT means at least that one of two assumptions fails. Bell' theorem and its improved versions only showed that LRT is not consistent with the QM's results, but they do not tell us that both locality or realism or both result in this inconsistency. Does there exists non-locality realism or non-realism locality? Recently, Simon Groblacher et al [11], based on Leggett's work [16], showed an important conclusion that

non-locality realism (i.e., give up locality but keep in the realism) is still inconsistent with QM's predictions by both theory and experiments. But this topic still need to be discussed further.

Although the two assumptions of LRT as shown above seem to be clear, they are abstract when they are applied to real experimental situations. This is the cause that one cannot judge easily whether or not there exists non-locality realism or non-realism locality according to the assumptions. In this work, we first discuss the detailed and operational description of realism, and show that any system being in a pure state or a non-maximally mixed state has property of non-realism. Then we present a strong and a weak description of locality, and discuss the connections between non-locality and entanglement and correlation. With the descriptions, one can easily find theoretically and experimentally the inconsistency of locality and realism with QM, and that both non-locality realism and non-realism locality are should be given up in QM. We also present experimental schemes feasible under current technologies to test the non-locality realism.

Let's first consider the realism. EPR said that "We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unit) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*". They added that "Regarded not a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality". EPR' criterion implies that the values of physical reality elements can be predicted with certainty without disturbing the systems. An observable should be a physical reality element. So according to EPR's criterion and the description of realism in some other papers [3, 10, 11, 17], the main idea of real-

ism can be expressed as follows: *for a definite pure state  $\psi$  of a system one can predict with certainty the value of any observable  $A$  without disturbing the system.*

If the state  $\psi$  is not the eigenstate of the observable  $A$ , QM claims that one can only predict the values  $A_1$  or  $A_2$  of quantity  $A$  with probability  $p_1$  or  $p_2$  (For simplicity, we suppose  $A$  has only two eigenvalues), respectively, and the measurements will disturb the system. This is not consistent with realism. To keep the realism, hidden-variable model was suggested. HVM accept the probability results of the measurement of observable  $A$  predicted by QM, but it thinks that the realism is true and our no knowledge of the hidden-variable is the cause of the probability results. According to the HVM, the property of the system depends on not only the state  $\psi$  but also a hidden-variable  $\lambda$  of which we now have no knowledge. It is to say that the system may be in state  $\psi(\lambda_1)$  or  $\psi(\lambda_2)$  with probability  $p_1$  or  $p_2$ , respectively. If the state of the system is  $\psi(\lambda_1)$  (or  $\psi(\lambda_2)$ ) one can predict with certainty the value  $A_1$  (or  $A_2$ ) of the observable  $A$  without disturbing the system, i.e.,

$$A\psi(\lambda_1) = A_1\psi(\lambda_1); \quad A\psi(\lambda_2) = A_2\psi(\lambda_2). \quad (1)$$

According to HVM, the average value of the observable  $A$  is  $\bar{A} = A_1p_1 + A_2p_2$ . According to QM, the average value  $\langle A \rangle = A_1p_1 + A_2p_2$ . So if we only consider one observable  $A$ , HVM and QM have the same prediction. It is to say that we cannot judge whether HVM is true or not by the prediction of the average value of one observable. Fortunately, we can judge HVM by investigating the joint prediction of two non-commuting observables  $A, B$  ( $[A, B] = C \neq 0$ ) as shown in the following Theorem 1.

**Theorem 1:** For any given state  $\psi$ , we can always find two non-commuting observables  $A$  and  $B$  such that the average values of the joint operators  $AB$  and  $BA$  can show the inconsistency between HVM and QM.

**Proof:** Without loss of generality, we introduce two hidden-variables  $\lambda$  and  $\lambda'$  to predict the values of observables  $A$  and  $B$ , respectively, where  $B\psi(\lambda'_1) = B_1\psi(\lambda'_1)$ ;  $B\psi(\lambda'_2) = B_2\psi(\lambda'_2)$  similar to the observable  $A$  in Eq. (1). We suppose that the system may be in the states  $\psi(\lambda_i, \lambda'_j)$  with probability  $p_{ij}$  ( $i, j = 1, 2$ ). According to HVM, we can have that

$$\begin{aligned} \overline{AB} &= \sum_{i,j} p_{ij} A_i B_j; \\ \overline{BA} &= \sum_{i,j} p_{ij} B_j A_i = \overline{AB} \end{aligned} \quad (2)$$

Since  $[A, B] = C \neq 0$ , Eq. (2) is not consistent with QM except for  $\langle \psi | C | \psi \rangle = 0$ . The left proof of the theorem 1 is to prove that for a given state  $\psi$ , one can always find two Hermitian operators  $A, B$  ( $[A, B] = C \neq 0$ ) such that  $\langle \psi | C | \psi \rangle \neq 0$ . To this end, we can choose

$$A = |\psi^\perp\rangle \langle \psi| + |\psi\rangle \langle \psi^\perp|; \quad (3)$$

$$B = -i(|\psi^\perp\rangle \langle \psi| - |\psi\rangle \langle \psi^\perp|); \quad (4)$$

$$C = 2i(|\psi^\perp\rangle \langle \psi^\perp| - |\psi\rangle \langle \psi|),$$

where  $|\psi^\perp\rangle$  is an arbitrary state orthogonal to  $|\psi\rangle$  of the system (The system can be of higher dimensionality, but the operators  $A, B$  and  $C$  only belong to the two-dimensional subspace of the whole space of the system). Thus,  $\langle \psi | C | \psi \rangle = -2i \neq 0$ , which is not consistent with HVM as shown in Eq.(2), and end the proof.

For a mixed state  $\rho$ , and two observables  $A, B$  ( $[A, B] = C \neq 0$ ), according to HVM we still have  $\overline{BA} = \overline{AB}$ . But by QM we have

$$\langle AB \rangle = \text{Tr}(\rho AB); \quad \langle BA \rangle = \text{Tr}(\rho BA).$$

Suppose that the eigen-decomposition of density matrix  $\rho$  is

$$\rho = \sum_{i=1}^n p_i |\psi_i\rangle \langle \psi_i|, \quad (5)$$

where  $|\psi_i\rangle$ s are orthogonal states and  $\sum_{i=1}^n p_i = 1, p_i \geq 0$ . If  $\rho$  is not the maximally mixed state  $\rho_{\max}$  ( $\rho_{\max} = \sum_{i=1}^n \frac{1}{n} |\psi_i\rangle \langle \psi_i|$ ),  $\rho$  can be expressed as  $\rho = \rho_{\max} + \sum_{i=1}^n \Delta p_i |\psi_i\rangle \langle \psi_i|$ , where  $p_i = \frac{1}{n} + \Delta p_i, \sum_{i=1}^n \Delta p_i = 0$ . Without loss of generality, let  $\Delta p_1 > 0, \Delta p_2 < 0$ , then for  $C = 2i(|\psi_1\rangle \langle \psi_1| - |\psi_2\rangle \langle \psi_2|)$ ,  $\text{Tr}(\rho C) = 2i(\Delta p_1 - \Delta p_2) \neq 0$ . So if we choose  $A = |\psi_2\rangle \langle \psi_1| + |\psi_1\rangle \langle \psi_2|$ ;  $B = -i(|\psi_1\rangle \langle \psi_2| - |\psi_2\rangle \langle \psi_1|)$ , then  $\text{Tr}(\rho AB) - \text{Tr}(\rho BA) = \text{Tr}(\rho C) \neq 0$  for  $\rho \neq \rho_{\max}$ , which show the inconsistency between HVM and QM. So we can have the following Theorem 2;

**Theorem 2:** For any non-maximally mixed state, we can always find two non-commuting observables  $A$  and  $B$  such that the average values of the joint operators  $AB$  and  $BA$  can show the inconsistency between HVM and QM.

If  $\rho = \rho_{\max} \propto I$ , then for any observables  $A$  and  $B$ ,  $\text{Tr}(\rho AB) = \text{Tr}(\rho BA)$ , which is consistent with QM. So we can conclude that *only the maximally mixed state always show the feature of realism.*

Theorem 1 and 2 can be tested by current experimental technologies. For example, let  $\rho = p|0\rangle \langle 0| + (1-p)|1\rangle \langle 1|$ , where  $|0\rangle, |1\rangle$  are eigenstates of the operator  $\sigma_z$ , i.e., spin-down and -up state or horizontal and vertical polarization states of photon, respectively. If we let  $A = \sigma_x$ ;  $B = \sigma_y$ , then we can test the Theorem 2 via measuring the average values of the operators  $\sigma_x \sigma_y$  and  $\sigma_y \sigma_x$  for the spin-1/2 particles or photon systems, where  $(\sigma_x, \sigma_y, \sigma_z)$  are Pauli operators.

We have shown the property of realism and its variance with QM in one system. But almost all known Bell Theorems involve two or more distant systems rather than one system. How to describe the realism of the composite system? Only if we regard the composite system as a whole system, and the state and the observables  $A, B$  in Eq. (3) are those of the whole system, then the description of realism and Theorems 1 and 2 can also work. The key is to find a set of observables  $A, B$  of the composite system which can be measured easily under present technologies.

Now we provide a feasible scheme under our technologies to show the realism's variance with QM in a bipartite system. The system under our consideration is two spin- $\frac{1}{2}$  particles (or two polarization photons) the state of which is  $\psi = \cos \alpha |00\rangle + \sin \alpha |11\rangle$ . let the observables  $A = \sigma_x^{(1)} \sigma_x^{(2)}, B = (\vec{n} \cdot \vec{\sigma}^{(1)})(\vec{n} \cdot \vec{\sigma}^{(2)})$ , where  $\vec{n} = (n_x, n_y, n_z)$  is a three-dimensional unit vector. For  $n_x = n_y = \frac{1}{\sqrt{2}}, n_z = 0, C = [A, B] = i(I \otimes \sigma_z^{(2)} + \sigma_z^{(1)} \otimes I)$ . According to HVM, the average values  $\overline{AB} = \overline{BA}$ , but according to QM,  $\langle AB \rangle - \langle BA \rangle = \langle \psi | C | \psi \rangle = i2 \cos 2\alpha$ . Only we choose  $\cos 2\alpha \neq 0$ , the difference of the expectations  $\langle AB \rangle$  and  $\langle BA \rangle$  can show the realism's variance with QM.

Let's now turn into locality. Locality claim that every measurement on A system does not affect instantaneously the state of B system if A and B are two distant systems. This means that measurements on the A system cannot change the state of the B system in limit time. A strong description (or a sufficient condition) of locality of a state  $\rho_{AB}$  in AB system can be expressed as: for *any measurement*, described by operators  $\{M_1, M_2, \dots, \sum_i M_i^+ M_i = I\}$ , on A system, whatever the outcome  $i$  is, the corresponding state  $\rho_{iB}$  of the B system is always  $\rho_B$ , where  $\rho_{iB} = \text{Tr}_A(M_i \rho_{AB} M_i^+)$  and  $\rho_B = \text{Tr}_A(\rho_{AB})$ . While a weak description (or a necessary condition) can be as: there exists a *measurement* on A system such that for each outcome  $i$  the state of the B system is always  $\rho_B$ .

Theorem 3: For a pure state  $\psi_{AB}$ , the system satisfies both the strong and the weak locality if and only if its state is separable.

Proof: If the state is pure and separable, the system satisfies obviously both the strong and the weak locality. If the state  $\psi_{AB}$  is entangled, then for any measurement denoted by operators  $\{M_1, M_2, \dots, \sum_i M_i^+ M_i = I\}$  on the A system there always exists a outcome  $i$  such that the B system will collapse correspondingly into a state not being  $\rho_B$  (except that  $M_i$  is proportional to a unit operator, which means the measurement is trivial). End of the proof.

For mixed states, some states only satisfy the weak description, but do not satisfy the strong description of

locality. For example, for the classical correlated state  $\rho_{AB} = \frac{1}{2} |00\rangle \langle 00| + \frac{1}{2} |11\rangle \langle 11|$ , if the measurement operators on the A system are  $M_1 = |0\rangle \langle 0|, M_2 = |1\rangle \langle 1|$ , the state of the B system will be collapsed into  $\rho_{1B} = |0\rangle \langle 0|, \rho_{2B} = |1\rangle \langle 1|$  corresponding to the outcome 1 and 2, respectively, which are not  $\rho_B = \frac{1}{2} |0\rangle \langle 0| + \frac{1}{2} |1\rangle \langle 1|$ . But if the measurement operators on the A system are  $M_1 = \frac{1}{2} |0+1\rangle \langle 0+1|, M_2 = \frac{1}{2} |0-1\rangle \langle 0-1|$ , both  $\rho_{1B}$  and  $\rho_{2B}$  are the same states as  $\rho_B$ . So the state  $\rho_{AB}$  only satisfies the weak description of locality.

Theorem 4 (Strong locality Theorem): The state  $\rho_{AB}$  of the AB system satisfies the strong description of locality if and only if the  $\rho_{AB}$  can be expressed as the form of  $\rho_{AB} = \rho_A \otimes \rho_B$ .

Proof: If  $\rho_{AB} = \rho_A \otimes \rho_B$ , it is obvious that the state  $\rho_{AB}$  satisfies the strong description of locality. If  $\rho_{AB} \neq \rho_A \otimes \rho_B$ , there exists correlation between the system A and B. So one can acquire some information of the B system via a measurement on the A system. This means that there is at least one measurement expressed by the operators  $\{M_1, M_2, \dots, \sum_i M_i^+ M_i = I\}$  on the

A system such that at least one state  $\rho_{iB}$  of the B system corresponding to measurement outcome  $i$  is not  $\rho_B$  (The acquired information of the B system via the measurement on the A system is  $S(\rho_B) - S(\rho_{iB})$ , where  $S(\cdot)$  is von Neumann entropy. If  $S(\rho_B) - S(\rho_{iB}) \neq 0$ , then  $\rho_{iB} \neq \rho_B$ ), and so the state  $\rho_{AB}$  does not satisfy the strong description of locality. End of the proof.

Theorem 4 shows any correlated states do not satisfy the strong description of locality.

For the weak description, we have following Theorem:

Theorem 5: Suppose A is a two-dimension system, and B is of arbitrary dimension. For a state  $\rho_{AB}$  of the AB system, if there exists a set of projective measurement expressed by operators  $\{P_{Ai}, i = 1, 2, \sum_{i=1}^2 P_{Ai} = I\}$  on A system such that the corresponding state of the B system is always  $\rho_B$  for each outcome  $i$ , then the state  $\rho_{AB}$  is separable.

Proof: Without loss of generality, we can imagine that AB is a subsystem of a big composite system ABC, the state of the composite system is

$$\psi_{ABC} = \sum_{k=1}^n \sqrt{p_k} |\psi_{AB}^k\rangle |k\rangle_C, \quad (6)$$

where  $|k\rangle_C$  is a set of orthogonal states of the C system,  $\{p_k, \psi_{AB}^k\}$  is a set of eigenstate-decomposition of the density matrix  $\rho_{AB}$ . If there exists a set of projective measurement operators  $\{P_{Ai} = |\phi_i\rangle \langle \phi_i|, i = 1, 2, \sum_{i=1}^2 P_{Ai} = I\}$  on A system such that the state of B system always is  $\rho_B$  for each outcome  $i$ , where  $\{|\phi_1\rangle, |\phi_2\rangle\}$  is a set of

bases of the A system, then  $\psi_{ABC}$  can be expressed as

$$\begin{aligned} \psi_{ABC} = & \sum_{i=1}^2 P_{Ai} \psi_{ABC} = \\ & \sqrt{\lambda_1} |\phi_1\rangle_A (|\eta_{11}\rangle_B |1\rangle_C + \dots + |\eta_{1n}\rangle_B |n\rangle_C) + \\ & \sqrt{\lambda_2} |\phi_2\rangle_A (|\eta_{21}\rangle_B |1\rangle_C + \dots + |\eta_{2n}\rangle_B |n\rangle_C) \end{aligned} \quad (7)$$

and for each outcome  $i$  the states of B system  $\rho_{Bi} = \sum_{k=1}^n |\eta_{ik}\rangle \langle \eta_{ik}|$ ,  $i = 1, 2$ , are equal to  $\rho_B$ , where  $|\eta_{ik}\rangle_B$ ,  $i = 1, 2; k = 1, \dots, n$ , are unnormalized vectors of the B system. Two sets of states  $\{|\eta_{i1}\rangle, \dots, |\eta_{in}\rangle\}$ ,  $i = 1, 2$ , are two sets of the pure-state-decompositions of the density matrix  $\rho_B$ , so they can be connected by a unitary matrix [18] as

$$[U_0] \begin{bmatrix} |\eta_{11}\rangle \\ \vdots \\ |\eta_{1n}\rangle \end{bmatrix} = \begin{bmatrix} |\eta_{21}\rangle \\ \vdots \\ |\eta_{2n}\rangle \end{bmatrix}, \quad (8)$$

where  $U_0$  is a unitary matrix. By linear algebra, any unitary matrix  $U_0$  can be diagonalized by a unitary matrix  $U$ , i.e.,

$$UU_0U^{-1} = \Lambda, \quad (9)$$

where  $\Lambda$  is a diagonal matrix and its diagonal elements  $\Lambda_{kk}$  ( $k = 1, \dots, n$ ) are of norm 1 (i.e.,  $|\Lambda_{kk}| = 1$ ). On the other hand, considering another set of bases of the C system  $\{|1'\rangle_C, \dots, |n'\rangle_C\}$  such that the transformation matrix from the bases  $\{|1\rangle_C, \dots, |n\rangle_C\}$  to bases  $\{|1'\rangle_C, \dots, |n'\rangle_C\}$  is  $U$ , we can re-express the state  $\psi_{ABC}$  as

$$\begin{aligned} \psi_{ABC} = & \sqrt{\lambda_1} |\phi_1\rangle_A (|\eta'_{11}\rangle_B |1'\rangle_C + \dots + |\eta'_{1n}\rangle_B |n'\rangle_C) + \\ & \sqrt{\lambda_2} |\phi_2\rangle_A (|\eta'_{21}\rangle_B |1'\rangle_C + \dots + |\eta'_{2n}\rangle_B |n'\rangle_C), \end{aligned} \quad (10)$$

where

$$[U] \begin{bmatrix} |\eta_{i1}\rangle \\ \vdots \\ |\eta_{in}\rangle \end{bmatrix} = \begin{bmatrix} |\eta'_{i1}\rangle \\ \vdots \\ |\eta'_{in}\rangle \end{bmatrix}, i = 1, 2. \quad (11)$$

Combining Eqs. (8, 9 11), we have that

$$\begin{bmatrix} |\eta'_{21}\rangle \\ \vdots \\ |\eta'_{2n}\rangle \end{bmatrix} = [\Lambda] \begin{bmatrix} |\eta'_{11}\rangle \\ \vdots \\ |\eta'_{1n}\rangle \end{bmatrix}.$$

So Eq. (10) can be expressed as

$$\begin{aligned} \psi_{ABC} = & (\sqrt{\lambda_1} |\phi_1\rangle_A + \Lambda_{11} \sqrt{\lambda_2} |\phi_2\rangle_A) |\eta'_{11}\rangle_B |1'\rangle_C + \\ & \dots + (\sqrt{\lambda_1} |\phi_1\rangle_A + \Lambda_{nn} \sqrt{\lambda_2} |\phi_2\rangle_A) |\eta'_{1n}\rangle_B |n'\rangle_C \end{aligned} \quad (12)$$

Obviously, from Eq. (12) we can see that the state  $\rho_{AB}$  is separable, and end the proof.

Theorems 3, 4 and 5 imply that the entangled states and correlated states might show the variance between locality and the QM's prediction.

In summary, we present the detailed and applicable descriptions of realism and locality in the real experimental situations. With the descriptions, one can easily find theoretically and experimentally the inconsistency of locality and realism with QM. We also present experimental schemes feasible under current technologies to test the non-locality realism. However, there are some open questions need to be discussed further. For example, what is the difference between the non-realism and the non-locality in a single particle as shown in Refs [19, 20, 21]?

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